

# On accuracy assessment of celestial reference frame VLBI realizations

Zinovy Malkin

Central (Pulkovo) Astronomical Observatory RAS,  
Pulkovskoe ch. 65-1, St. Petersburg 196140, Russia  
malkin@gao.spb.ru

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## Abstract

In this paper, we propose to use the scatter of celestial pole offset (CPO) series obtained from VLBI observations as a measure of the accuracy of the celestial reference frame (CRF) realizations. Several scatter indices (SIs) including those proposed for the first time are investigated. The first SI is based on analysis of residuals of CPO series w.r.t. Free Core Nutation (FCN) model. The second group of SIs includes Allan deviation and its extensions, which allow one to treat unequal and multidimensional observations. Application of these criteria to several radio source catalogues showed their ability to perform a preliminary assessment of the quality of radio source catalogues, and 2D Allan deviation estimate seems to be a most sensitive measure. However, in common case, the sensitivity of tested criteria is yet not sufficient to distinguish between radio source catalogues of the highest quality. Proposed extensions of Allan deviation, weighted and multidimensional, can be effectively used also for statistical analysis of astro-geodetic and other time series.

## 1 Introduction

Very long baseline interferometry (VLBI) is the base technique provided International Celestial Reference Frame (ICRF) realized as a set of radio source coordinates (Ma et al. 1998). Improvement of the ICRF accuracy is one of the primary task for International VLBI Service for Astrometry and Geodesy (IVS), and assessment of the ICRF accuracy is of primary importance in its improvement (Schlüter et al. 2002).

Many studies of the accuracy of the ICRF have been performed during last years. Some authors have investigated the time behavior of radio source position, e.g. Gontier et al. (2001), Feissel-Vernier (2003). Others have studied the accuracy of different CRF realizations by means of investigation of stability of the coordinate axis, e.g. Arias (1988), Arias (2004). However, existing method allow us to investigate only *differences* between Celestial Reference Frame (CRF) realizations, or, in other words, radio source position catalogues. There is no evident and received method to assess the *absolute* ICRF accuracy.

In this paper, we consider possible criteria, which allow one to assess the accuracy of radio source catalogues from its impact on results of determination of the celestial pole offset (CPO) from VLBI observations. We propose to use the scatter of CPO time series obtained from VLBI observations as a measure of the accuracy of the CRF realizations. Several scatter indices (SIs), including those proposed for the first time are investigated. The first SI is based on analysis of residuals of CPO series from a reference series obtained as the IAU2000A precession-nutation model with addition of the Free Core Nutation (FCN) contribution. The second group of SIs includes Allan deviation (ADEV) and its extensions. Two such extensions are proposed. The first one, weighted Alan variance WADEV, which allow one to treat unequal measurements, which we mostly deal with in geodesy. The second extension of the classical definition is weighted multidimensional Allan deviation estimator WMADEV, which can be applied to combined analysis of several associated time series.

We have applied these SIs to several radio source position catalogues. The tested catalogues of the first group were obtained in the framework of the joint pilot project of the International Earth Rotation and Reference Systems Service (IERS) and the IVS (Ma 2004, Call for Participation). The second group of catalogues consists of the latest submissions of IVS Analysis Centers to the IVS. The latest ICRF realization, ICRF-Ext.2 (Fey 2004), was also tested in this study. Based on the results of this work, we can make a conclusion on ability of various SIs to detect difference in accuracy (quality) of the CRF realizations provided by VLBI Analysis Centers.

## 2 Scatter indices

Investigation of the scatter of a time series of geodetic, geodynamical or other data under interest is proven to be a powerful tool for assessment of its quality and statistical characteristics. The scatter of the time series can be computed in different way. We will use the term "scatter index" (SI) to distinguish the results obtained by different methods. Advantage of using SIs for investigation of time series is its independence, in the most of cases, of systematic errors in tested results, trends and low-frequency variations.

### 2.1 Residuals in CPO series

Both precision and accuracy of the CPO estimates directly depend on accuracy of radio source positions used during VLBI data processing. There is no external *absolutely accurate* reference series that may be used for comparison, for only VLBI technique provides the highly-accurate CPO measurements.

The scatter of Earth Orientation Parameters (EOP) time series is widely used for many years for comparison of EOP results obtained in different analysis centers. We will try to apply it to comparison of CRF realization. Similar test was used in Feissel-Vernier et al. (2006), where the authors compared several radio source catalogues obtained in the course of their work using the WRMS differences with the IAU2000 precession-nutation model. However, it seems that such a test can be improved by using of the observed correction to the IAU2000 model, the contribution of the Free Core Nutation in the first place. In such a case, the residuals between VLBI nutation series and model becomes much less, and the

test, hopefully, will be more sensitive. Thus we used the IAU2000A model with addition of the FCN contribution as the reference series. For our purpose, we computed the FCN contribution by smoothing of the differences between the IVS combined EOP series and the IAU2000A model.

However, this SI cannot be considered as really independent estimate of the CPO scatter since any FCN model derived from a comparison of observations with theory depends on results of VLBI data processing. Evidently, such an estimate implicitly depends on underlying CRF realizations. Hence, FCN criterion provides only comparison of tested radio source catalogue realization with some averaged CRF realization.

## 2.2 Allan deviation

Allan variance estimator was originally developed for investigation of noise parameters of frequency standards. In the last years, Allan-variance-based techniques is often applied to analysis of geodetic and geodynamical time series. Usually, square root of Allan variance ADEV is used for this purpose. IERS EOP Product Center employs this technique for assessment of the statistical characteristics of the EOP series (Gambis 2002). ADEV estimates are used for investigation of stochastic properties of station coordinates time series, e.g. Malkin & Voinov (2001), Le Bail & Feissel-Vernier (2003), Feissel-Vernier & Le Bail (2005), Le Bail (2006). Feissel-Vernier (2003) has also used Allan variance for analysis of the noise in radio source position time series.

Allan deviation estimate for time series  $y_1, y_2, \dots, y_n$  is given by

$$ADEV = \sqrt{\frac{1}{2(n-1)} \sum_{i=1}^{n-1} (y_{i+1} - y_i)^2}. \quad (1)$$

Commonly speaking, this formula is valid for equal measurements. However, in geodetic and astrometric practice, we usually deal with unequal measurements. In this case, we have the measurements with associated uncertainties  $s_1, s_2, \dots, s_n$ . To treat unequal data, the following extension of ADEV can be proposed.

$$WADEV = \sqrt{\frac{1}{2p} \sum_{i=1}^{n-1} p_i (y_{i+1} - y_i)^2}, \quad (2)$$

$$p = \sum_{i=1}^{n-1} p_i, \quad p_i = \frac{1}{s_i^2 + s_{i+1}^2}.$$

Fig. 1 shows an example of actual series of station coordinates, which show that the difference between ADEV and WADEV values may be quite substantial.

Further extension of WADEV estimator is useful for analysis of some types of time series. When the statistical parameters of astro-geodetic time series are investigated, we, in fact, often deal with multidimensional values, e.g. terrestrial coordinates and/or velocity (3D or 6D), celestial coordinates and/or proper motion (2D or 4D), and so on. In such a case, we can use multidimensional notation  $y_i = (y_i^1, y_i^2, \dots, y_i^k)$  with the associated k-dimensional uncertainties  $s_i = (s_i^1, s_i^2, \dots, s_i^k)$ . Thus, we can define the k-dimensional Allan deviation as

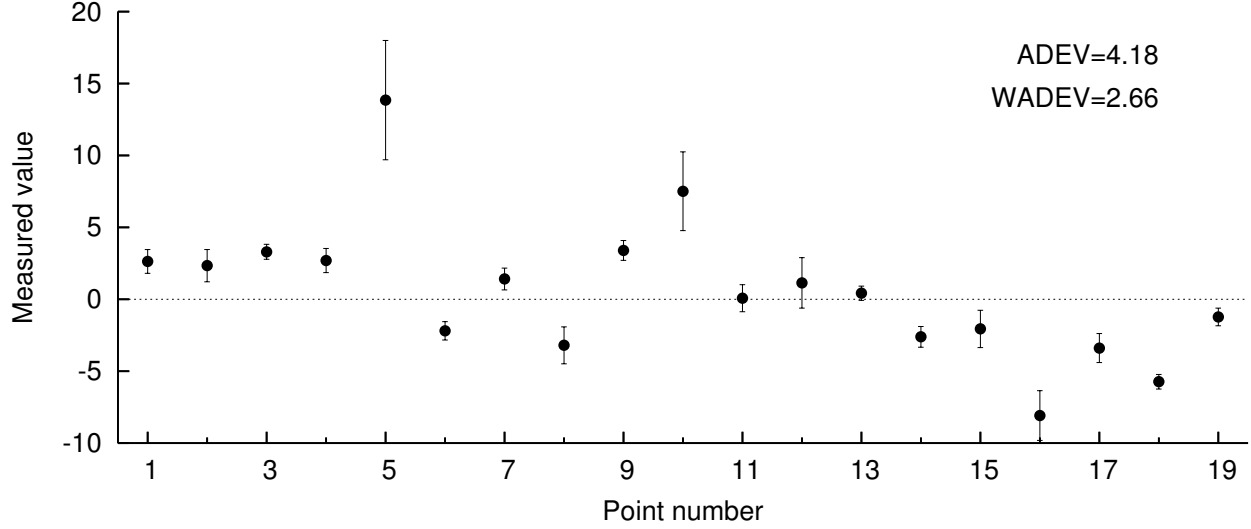


Figure 1: An example of estimation of classical (ADEV) and weighted (WADEV) Allan deviation for unequal time series.

$$\begin{aligned}
 WMADEV &= \sqrt{\frac{1}{2p} \sum_{i=1}^{n-1} p_i |y_{i+1} - y_i|^2}, \\
 p &= \sum_{i=1}^{n-1} p_i, \quad p_i = \frac{1}{\sum_{j=1}^k [(s_i^j)^2 + (s_{i+1}^j)^2]}.
 \end{aligned} \tag{3}$$

It is needless to say that both WADEV and WMADEV estimates can be easily generalized, like classical ADEV estimate, for different sampling interval as well as overlapping intervals.

### 3 Comparison of catalogues

For our study, we computed several CPO time series with the same processing options, except different underlying radio source catalogues. We used 504 R1 and R4 VLBI sessions observed in the period 2002.0–2007.0. Then we compute the SI described above for all the series.

We included in this comparison three groups of VLBI CRF realizations. First, we used eight radio source catalogues obtained in the framework of the joint IERS-IVS Pilot Project mentioned above. The catalogues were provided by eight IVS Analysis Centers: AUS (Geoscience Australia), BKG (Bundesamt für Kartographie und Geodäsie, Germany), DGF (Deutsches Geodätisches Forschungsinstitut, Germany), GSF (NASA Goddard Space Flight Center, USA), JPL (Caltech/NASA Jet Propulsion Laboratory, USA), MAO (Main Astronomical Observatory of National Academy of Sciences of Ukraine), SHA (Shanghai Astronomical Observatory, China), USN (U. S. Naval Observatory, USA). Each Analysis Center

Table 1: Scatter indices for celestial pole offset series obtained with different radio source catalogues. See text for details. Unit:  $\mu\text{as}$ .

Catalogue	FCN	ADEV	WADEV	WMADEV
Pilot Project catalogues, 1979–2005				
AUS1	99	116	108	153
BKG1	96	111	104	147
DGF1	96	115	104	148
GSF1	96	111	103	146
JPL1	98	113	105	149
MAO1	96	113	104	147
SHA1	96	114	104	148
USN1	96	114	104	147
Pilot Project catalogues, 1990–2005				
AUS2	97	113	104	148
BKG2	95	112	103	146
DGF2	96	111	103	146
GSF2	95	111	103	146
JPL2	96	110	102	145
MAO2	95	111	103	145
SHA2	96	112	104	147
USN2	95	111	102	145
Latest catalogues				
AUS2006B	97	112	103	147
BKG2006C	96	112	103	146
CGS2006A	95	110	102	144
GSF2007A	96	111	103	146
IAA2006A	97	114	106	150
USN2006C	95	110	102	144
ICRF-Ext.2	99	120	111	157

has submitted two version of catalogue. The first one was computed using all available VLBI data. For the second version, only the observations made since 1990 have been used. Then we included in our study six radio source catalogues submitted by IVS Analysis Centers in 2006–2007 in the framework of routine operations. The latest ICRF realization, ICRF-Ext.2 (Fey 2004), was also used in our study.

Results of this test are presented in Table 1. In the table, FCN column shows the WRMS differences between CPO series computed with given radio source catalogue and IAU2000A+FCN reference series, ADEV and WADEV columns contain classical and weighted Allan variance estimates. The values presented in the FCN, ADEV and WADEV columns are computed as the mean of the estimates for  $X$  and  $Y$  celestial pole coordinates. The WMADEV column contains 2D estimates computed with (3),  $k = 2$ , for both pole coordinates.

One can see that the accuracy of all the compared catalogues as estimated by this method is close to each other. Nevertheless, some discrepancies can be clearly seen. In particular,

the ICRF-Ext.2 showed the worst result, maybe because ITRF-Ext.2 212 *defining* source positions were saved from the first ICRF. This can be mentioned that, in the analyzed data, 186 *defining* sources were observed of total number of 507 (*i.e.* 36.6%), and there were 307155 observations of *defining* sources of total number of 953637 (*i.e.* 32.2%). If so, it might be inadvisable to keep coordinates of the *defining* ICRF sources in the successive ICRF updates.

The organizers of the IERS-IVS Pilot Project requested from the participating Analysis Centers to submit two versions of catalogues obtained using all available data (version 1) and data from 1900 only (version2). The intention was to investigate an impact of data selection on the quality of CRF realization. From our test, one can see that catalogues of version 2 show small but steady improvement of the accuracy. This effect can be also explained by the greater weight of R1/R4 sessions in the catalogues of version 2 w.r.t. catalogues of the version 1 though. On the other hand, previous studies (Malkin 2004a,b) have shown that both EOP and precession parameters derived from VLBI observations show much less uncertainty when only data from 1990 are used for analysis.

As to comparison of different SIs considered in this test, 2D Allan variance seems to be the most sensitive criterion. It is advisable to use the weighted multidimensional Allan deviation for investigation of other astro-geodetic multidimensional time series.

## 4 Conclusion

In this paper, we tested a possibility of using the scatter of celestial pole offset (CPO) series obtained from VLBI observations as a measure of the accuracy of the CRF realizations. Several scatter indices (SIs), including those proposed for the first time are investigated.

The first SI is based on analysis of residuals of CPO series w.r.t. IAU2000A precession-nutation model supplemented with the FCN contribution. It turned out to be less sensitive than other tested SIs. Besides this SI cannot be considered as fully independent since both IAU2000A model and FCN contribution depend on previous VLBI data analysis, in particular, used CRF realizations. Conversely, the SIs based on Allan deviation and its extensions proposed in this paper provide independent estimate of the quality of CRF realizations.

Application of these criteria to several radio source catalogues has shown their ability to perform a preliminary assessment of the quality of radio source catalogues, and 2D Allan deviation estimate seems to be a most sensitive measure. However, the sensitivity of tested criteria is yet not sufficient to distinguish between radio source catalogues of the highest quality.

Proposed extensions of the Allan deviation estimator, weighted WADEV and multidimensional WMADEV, can be effectively used also for statistical analysis of geodetic and other time series, *e.g.* Earth Orientation Parameters and station position time series.

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